FRACTALS - 7

**Detrended Cross Correlation Analysis (DCCA)**

Detrended cross-correlation analysis (DCCA) was recently introduced by Podobnik and Stanley [1], and is designed to analyze power-law cross-correlations between two simultaneously recorded non-stationary time series. The DCCA method has been successfully used in the analysis of physiological signals [2,3], traffic time series [4,5] climatic data [6,7] financial data [8,9,10] and human behavior records [11,12].

It proceeds as follows:

1. Two simultaneously recorded time series and are integrated to produce and, where  is an integer between 1 and.
2. Next, the integrated series are divided into segments of equal length and a linear (or higher order polynomial) regression is performed for each segment to capture the local trend.
3. The integrated series and are detrended by subtracting the local trends and (ordinates of the straight line or polynomials within each segment from the data in each segment.
4. The detrended covariance is calculated as



1. Repeating this calculation for all segment sizes provides the relationship between and the segment size. If original series are power-law cross-correlated, then



and the scaling exponent is determined from the linear regression of  versus .

The interpretation ofis similar to that of the DFA exponent. Long-term cross-correlations between two series imply that each series has long memory of its previous values, as well as a long memory of the previous values of the other series. When only one series is analyzed, the DCCA method is equivalent to the Detrended fluctuation analysis (DFA) method [1].

http://www.sciencedirect.com/sd/blank.gif**Example**

We apply DFA and DCCA on absolute values of daily price changesand daily volume changesof Bovespa index, whereand  are daily returns of price and volume , respectively, and andare standard deviations of  and over the duration of time series.



Figure 1. DFA and DCCA graphs for absolute values of price changes and volume changes for daily returns of Bovespa index.

Table 1. DFA exponents and DCCA exponent of price changes  and volume changes for daily returns of Bovespa index

|  |  |  |
| --- | --- | --- |
| DFA price | DFA volume | DCCA |
|  |  |  |
|  |  |  |

**DCCA cross-correlation coefficient**

 It was shown recently  [[13,14]](http://www.sciencedirect.com/science/article/pii/S0378437115000059#br000235) that the relationship between DFA exponents  and  of two auto-correlated series, and the corresponding DCCA exponent , may be established using the cross-correlation coefficient



If the two series are not cross-correlated  oscillates about zero (bounded between −1 and 1), for anti cross-correlated series  is negative, for positively cross-correlated series   is positive and , such that

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